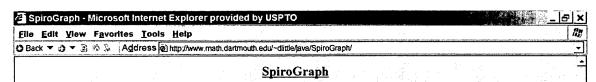


Claims 1 and 6.



The path of a point fixed relative to a circle that rolls along a straight line is called a prochord. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is iding on a level street. The reflector rotates around the but of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an applet that demonstrates this (without the bicycle) (NOTE: As of 10-800, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems.)

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where Rr, and p are defined below. The applet below allows you to create all the SpiroGraphu your heart desires by varying the values of R, s and p, as well as the following parameters:

- Radius 1 (R)
  Radius of circle (equator) centered at the origin.
  Radius of circle (hequator) centered at the origin.
  Radius of circle (hicycle wheel) centered at (R=\(\tau\))
  Position (p)
  Detaunce of Point (reflector) from the center of Circle 2, the circle of radius r.

Distance of Point (reflector) from the center of Circle2, the circle of radius s.

Velectly
Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not:

Resolution
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points (a(t), y(t)) are plotted for t=0, 1, 2, 3, 4, 5, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 8, 10, ..., 255, 360 are used. Be careful, the larger the Resolution, the longer it will take to draw the SpiroGraph. Normally, a value between 270 and 360 will be good enough. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraphs.

Hilde/Show
Chamoline the value of this button will determine whether or not to display the circles/axes while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Hide" the circles/axes.

- Hilds/Show

  Changing the value of this button will determine whether or not to display the circles/axes while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Hide" the circles/axes.

  Clear

  This button will clear the screen of all SpiroGraphs.

- This button will clear use screen or an open consequence.

  Reset

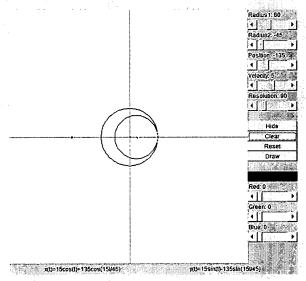
  This button stops the cument SpiroGraph and returns Circle? and the Point to its original position

  DrawPause/Resuma

  These buttons should be self explanatory.

  Red/Green/Blue

  You can adjust the visites of these sliders to draw your SpiroGraph in virtually any color you like!



SIMPLE	Radiusl	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Very the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved rose	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	.30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	50	-45	-101	270	How about a rounded triangle or pentagon?
Gold fish	75	-25	85	270	This is supposed to be fun, right?

### SpiroGraph - Microsoft Internet Explorer provided by USPTO

File Edit Yiew Favorites Iools Help

O Back ▼ 🖒 ▼ 🚉 🖒 📜 Address 🗃 http://www.math.dartmouth.edu/-dlittle/java/SpiroGraph/

### **SpiroGraph**

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The path of a point fixed relative to a circle that rolls along a straight line is called a prochord, The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is riding on a level street. The reflector rotates around the hub of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an <u>applet</u> that demonstrates this (without the bicycle) (NOTE: As of 10/600, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems.)

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x(t)=(R+r)cos(t) - p\*cos((R+r)t/r) y(t)=(R+r)tin(t) - p\*sin((R+r)t/r)

where R.r. and p are defined below. The applet below allows you to create all the SpiroGraphs your heart desires by varying the values of R, 1 and p, as well as the following parameters:

- Radius I (R)
   Radius of circle (equator) centered at the origin.
- Radius 2 (r)

  Radius 2 (r)

  Radius of circle (bicycle wheel) centered at (R-r,0)

- Posicion (p)

  Distance of Point (reflector) from the center of Circle2, the circle of radius s.

Distance of Point (reflector) from the center of Circle2, the circle of radius r.

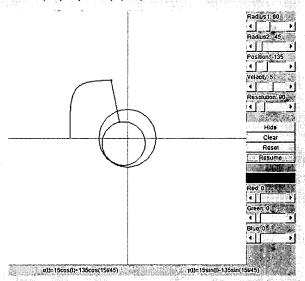
Velocity
Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastesst. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

Resolution
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 500, the points (a(t), y(t)) are plotted for t=0, 1, 2, 3, 4, 5, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 355, 360 degrees. With a Resolution of 180, only the values of controls. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 350 will be good enough. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 350 will be good enough. With a tmall value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraph. Normally, a value between 270 and 350 will be good enough. With a tmall value of Radius 2 (say Radius 2=1), a small change in Resolution of remarkably different SpiroGraph. Sometimes and states a small change in Resolution of 180, only the small value of Radius 2 (say Radius 2=1), a small change in Resolution of 180, only the small value of Radius 2 (say Radius 2=1), a small change in Resolution of 180, only the small value of Radius 2 (say Radius 2=1), a small change in Resolution of 180, only the small value of Radius 2 (say Radius 2=1), a small change in

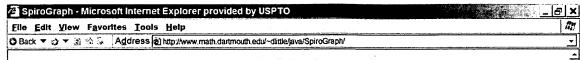
- Reset
  This button stops the current SpirtoGraph and returns Circle2 and the Point to its original position
  DrawPauseResume

  DrawPauseResume
- These buttons should be self explanatory.
  Red/Green/Blue

Voicen adjust the values of these aliders to draw your SpiroGraph in virtually any color you like!



					1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
SIMPLE CURVES	Radiusl	Radius2	Position	Resolution	Try this:
Cardioid	50	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	50	-13	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 oz 6 cusps. Also, vary the position and see what happens.
Four-leaved rose	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	.39	.30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	45	-101	270	How about a counded triangle or pentagon?
Gold fish	75	-25	85	270	This is supposed to be fun, right?
rose Vertical Line Ellipse Rounded Square	60 60	-30 -30	-90 -101	270 270 270	vary the position and see what happens.  Try to vary the radius and position to get 3,5 or What would you change to get a horizontal line What would you have to change in order to get omerade horizontally? How would you get an el inside of the fixed circle? How about a rounded triangle or pentagon?



The path of a point fixed relative to a circle that rolls along a straight line is called a prochold. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is riding on a level street. The reflector rotates around the lub of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an <u>applet</u> that demonstrates this (without the bicycle) (NOTE: As of 10/650), I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know a you experience any further problems.)

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#(t)=(R+r)cos(t) - p\*cos((R+r)t/r)
y(t)=(R+r)cin(t) - p\*sin((R+r)t/r)

where R.1, and p me defined below. The applet below allows you to create all the Spinofraphs your beart desires by varying the values of R, a and p, as well as the following parameters:

- Radius 1 (R)
   Radius of circle (equator) centered at the origin.
   Radius 10
   Radi

- Changing the value of this button will determine whether or not to display the circles/axes while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Hide" the circles/axes. Clear
   This button will clear the screen of all SpiroGraphs.

- This button will clear the screen of all SpiroGraphs.

  Reset

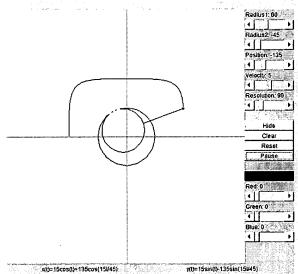
  This button stops the cument SpiroGraph and returns Circle2 and the Point to its original position

  DrawPauseRessum

  These buttons should be self explanatory.

  RedGreen/Blue

  Von care adjust the values of these slifters to draw your SpiroGraph in virtually any color your bled.
- - You can adjust the values of these sliders to draw your SpiroGraph in virtually any color you like!



SIMPLE	Radnisl	Radius2	Position	Resolution	Try this:	
Cardioid	60	60	60	279	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?	
Astroid	60	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.	
Four-leaved	60	-15	45	270	Try to vary the radius and position to get 3,5 or 6 leaves	
Vertical Line	60	-30	-30	270	What would you change to get a horizontal line?	
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?	
Rounded Square	60	-45	-101	270	How about a rounded triangle or pentagon?	
Gold fish	75	-25	85	279	This is supposed to be fun, right?	



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Now think about what happens if the circle is instead rotating around another circle, in other words, the cyclist is now pedaling his her way around the equator, instead of down the street. Math an epicycloid. The rest of the world calls them SpicoGraphs!! The parametric equations for these curves are given by:

x(t)=(R+r)ccs(r) - p\*cos((R+r)r/r) y(r)=(R+r)sin(t) - p\*sin((R+r)r/r)

387.4m 3 3 5 5 5 5

where R., and p are defined below. The applet below allows you to create all the SpiroGraphs your heart desires by varying the values of R, a and p, as well as the following parameters:

- Radius 1 (R)
   Radius of circle (equator) centered at the origin.
   Radius 2 (r)
   Radius 2 (r)
   Radius 2 (r)
- Position (p)
   Distance of Point (reflector) from the center of Circle2, the circle of redius s.

Distance of Point (reflector) from the center of Circle2, the circle of radius r.

Velocity
Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

Resolution
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points (a(t), y(t)) are plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of the value of Radius 2 (say Radius 2 1), a small change in Resolution can result in remarkably different SpiroGraphs. Normally, a value between 270 and 360 will be good enough. With a small value of Radius 2 (say Radius 2 1), a small change in Resolution can result in remarkably different SpiroGraphs are drawn much faster if you "Hide" the circles/ases.

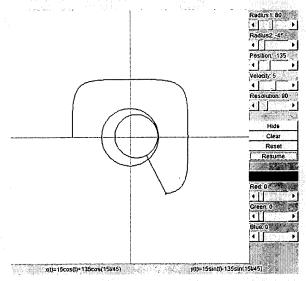
Clear

This botton will clear the screen of all SpiroGraphs.

Reset

- Reset
   This button stops the current SpiroGraph and returns Circle2 and the Point to its original position
   Draw/Pause/Resume

You can adjust the values of these sliders to draw your SpiroGraph in virtually any cotor you like!



SIMPLE	Radiusl	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the night. How could you get a cardioid oriented differently?
Astroid	50	-13	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	-30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	279	How about a counded triangle or pentagon?
Gold fish	75	-25	85	270	This is supposed to be fun, right?

Ay

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The path of a point fixed relative to a circle that rolls along a straight line is called a *wochold*. The easiest way to visualize this phenomenon is to think of the path of a reflector on a bicycle as someone is noting on a level street. The reflector rotates around the hub of the wheel, but yet the hub of the wheel is moving relative to the ground. Here is an <u>applet</u> that demonstrates this (without the bicycle) (NOTE: As of 10:600, I have rewritten the cycloid applet. I had received some comments suggesting that the old version caused some computers to crash. Hopefully the problem has been fixed. Let me know if you experience any further problems.)

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where R1, and p are defined below. The applet below allows you to create all the SpiroGraphs your heart denies by varying the values of R, 1 and p, as well as the following parameters:

- Radius 1 (R)
   Radius of circle (equator) centered at the origin.
   Radius 2 (r)
   Radius of circle (bicycle wheel) centered at (R+1,0)

Velectiv
 Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted while drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!
 Resolution

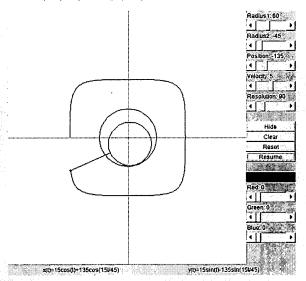
Resolution
 Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of 360, the points (a(t), y(t)) me plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of 180, only the value of two the SpiroGraph. Normally, a value between 200 and 350 will be good enough. With a small value of Radius 2 (say Radius 2-1), a small change in Resolution can result in remarkably different SpiroGraphs.
 Hide-Show
 Changing the value of this button will determine whether or not to display the circles/saces while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Hide" the circles/saces.

- e Reset
  This button was crear the screen of an Spirocraphs.

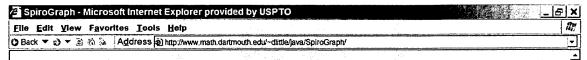
  Reset
  This button stops the cuneral SpiroGraph and returns Circle2 and the Point to its original position
  DrawPause/Ressume
  These buttons should be self explanatory.

  Red/Green/Blue

You can adjust the values of these sliders to diam your SpiroGraph in virtually any color you like!



SIMPLE CURVES	Rediusl	Radius2	Position	Resolution	Try this:
Cardioid	60	60	60	270	See what happens when you vary the postion to the left or to the right. How could you get a cardioid oriented differently?
Astroid	50	-15	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved	į 60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	.30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	, 60	-45	-101	270	How about a counded triangle or pentagon?
Gold fish	75	-25	85	270	This is supposed to be fun, right?



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y(t)=(R\*r)cin(t) - p\*cin((R\*r)t/r)

where R.r. and p are defined below. The applet below allows you to create all the SpiroGraphs your heart desires by varying the values of P., a and p, as well as the following parameters:

- Radius 1 (R)
  Radius of circle (equator) centered at the origin.
  Radius of circle (nicycle wheel) centered at (R→,0)
  Position (p)
  Distance of Point (reflector) from the center of Circle 2, the circle of radius s.

Distance of Point (reflector) from the center of Circle2, the circle of radius r.

Velocity
Speed at which the SpiroGraph is drawn, with 0 being the slowest and 10 being the fastest. Can be adjusted white drawing is taking place. Sometimes half the fun is seeing your SpiroGraph being drawn! Sometimes not!

Resolution
Controls how precise the SpiroGraph is drawn. For instance, with a Resolution of \$60, the points {x}(x(t), y(t)) are plotted for t=0, 1, 2, 3, 4, 5, ..., 359, 360 degrees. With a Resolution of \$80, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of \$80, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of \$80, only the values of t=0, 2, 4, 6, 3, 10, ..., 359, 360 degrees. With a Resolution of \$80, only the tonger it will take to draw the SpiroGraph. Normally, a value between 270 and \$80 will be good enough. With a small value of Radius 2 (say Radius 2=1), a small change in Resolution can result in remarkably different SpiroGraph.

\*\*HilderShow\*\*

\*\*HilderShow\*\*

\*\*Chamoins the value of this button will determine whether or not to display the circles/axes while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Tide" the circles/axes.

- Hide/Show
   Changing the value of this button will determine whether or not to display the circles/sacs while drawing is taking place. Note: SpiroGraphs are drawn much faster if you "Fide" the circles/sacs.
   Clear
   This button will clear the screen of all SpiroGraphs.

- This button will clear the screen of all SpiroGraphs.

  Reset

  This button stops the current SpiroGraph and returns Circle? and the Point to its original position

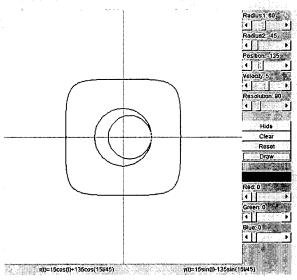
  DrawPause/Resuma

  These buttons should be self explanatory.

  Red/Green/Blue

  A continue of these a different of these a lifest to draw your SpiroGraph in virtually any color to that

You can adjust the values of these sliders to draw your SpiroGraph in virtually any cotor you like!



SIMPLE CURVES	Radiusl	Radius2	Position	Resolution	Try this:
Cardioid	50	60	60		See what happens when you vary the postion to the left or to the night. How could you get a cardioid oriented differently?
Astroid	60	-13	-15	270	Another name for an astroid is a hypocycloid of four cusps. What would you change to get the same picture rotated 45 degrees? Vary the radius and try to get 3,5 or 6 cusps. Also, vary the position and see what happens.
Four-leaved	60	-15	45	270	Try to vary the radius and postion to get 3,5 or 6 leaves
Vertical Line	60	-30	-30	270	What would you change to get a horizontal line?
Ellipse	50	-30	-90	270	What would you have to change in order to get an ellipse oriented horizontally? How would you get an ellipse on the inside of the fixed circle?
Rounded Square	60	-45	-101	270	How about a rounded triangle or pentagon?
Gold fish	j 75	~25	85	270	This is supposed to be fun, right?